Aufgabe 1

$$z = ax^2 + 2xy + 2ay^2$$

$$\frac{\partial z}{\partial x} = 2ax + 2y = 0$$

$$\frac{\partial z}{\partial y} = 2x + 4ay = 0$$

Gaußsches Eliminationsverfahren

x	y		
$2a$	2	0	
$\boxed{2}$	$4a$	0	$:2$
$2a$	2	0	$+$
$\boxed{1}$	$2a$	0	$\cdot(-2a)$
0	$\boxed{2-4a^2}$	0	$:(2-4a^2)$
1	$2a$	0	Vorauss.: $4a^2 \neq 2$
			d.h. $a^2 \neq \frac{1}{2}$
0	1	0 $\cdot(-2a)$	d.h. $a \neq \pm\frac{1}{\sqrt{2}}$
1	$2a$	0 $+$	
0	1	0	
1	0	0	

Also genau eine Lösung falls $a \neq \pm\frac{1}{\sqrt{2}}$

Spezialfall $a = \pm \frac{1}{\sqrt{2}}$

x	y	
~~0~~	~~0~~	~~0~~
1	$2a$	0
1	$2a$	0

Also hier unendlich viele Lösungen.

Aufgabe 2

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 3 & a & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \boxed{2} & 0 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ \\ \\ :2 \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 3 & a & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \boxed{1} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1/2 \end{array}\right) \begin{array}{l} \\ + \\ \cdot(-a) \\ \\ \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 1 & -a & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1/2 \end{array}\right) \begin{array}{l} \\ :3 \\ \\ \\ \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & \\ 0 & 0 & 1 & 0 & \\ 0 & 0 & 0 & 1 & \end{array}\right. \underbrace{\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1/3 & -a/3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/2 \end{array}}_{\text{Inverse der Matrix}}$$

Aufgabe 3

Notwendige Bedingung:

Produktregel:

$$y' = x^4 \frac{1}{x} + 4x^3 \ln x$$

$$= x^3 + 4x^3 \ln x$$

$$= x^3 (1 + 4 \ln x) = 0$$

Da $x > 0$ also

$$1 + 4 \ln x = 0$$

$$\ln x = -\frac{1}{4}$$

$$x = e^{-\frac{1}{4}}$$

Hinreichende Bedingung:

$$y'' = x^3 \left(4 \cdot \frac{1}{x}\right) + 3x^2 (1 + 4 \ln x)$$

$$= 4x^2 + 3x^2 (1 + 4 \ln x)$$

$$= x^2 (4 + 3 + 12 \ln x)$$

$$= x^2 (7 + 12 \ln x)$$

An der Stelle $x = e^{-\frac{1}{4}}$

$$y''(e^{-\frac{1}{4}}) = (e^{-\frac{1}{4}})^2 (7 + 12 \cdot \ln e^{-\frac{1}{4}})$$

$$= e^{-\frac{1}{2}} (7 + 12 \cdot (-\tfrac{1}{4}) \ln e)$$

$$= e^{-\frac{1}{2}} (7 - 3 \cdot 1)$$

$$= 4 e^{-\frac{1}{2}} > 0$$

$$y(e^{-\frac{1}{4}}) = (e^{-\frac{1}{4}})^4 \ln e^{-\frac{1}{4}}$$

$$= e^{-1} (-\tfrac{1}{4}) \ln e$$

$$= -\frac{1}{4} e^{-1} = -\frac{1}{4e}$$

Also Minimum im Punkt

$$\left(e^{-\frac{1}{4}} ; -\frac{1}{4e}\right)$$

Aufgabe 4

Substitution $g = x^2$

also $\frac{dg}{dx} = 2x$

also $dx = \frac{dg}{2x}$

Damit

$$\int x \sin x^2 \, dx = \int x \sin g \frac{dg}{2x}$$

$$= \frac{1}{2} \int \sin g \, dg$$

$$= \frac{1}{2} (-\cos g) + C$$

$$= -\frac{1}{2} \cos x^2 + C$$

und so

$$\int_0^a x \sin x^2 = \left[-\frac{1}{2} \cos x^2 \right]_0^a$$

$$= -\frac{1}{2} \left[\cos a^2 - \cos 0 \right]$$

$$= -\frac{1}{2} \left[\cos a^2 - 1 \right] = \frac{1}{2}$$

d.h.

$$\cos a^2 - 1 = -2 \cdot \frac{1}{2} = -1$$

$$\cos a^2 = 0$$

z.B.

$$a^2 = \frac{\pi}{2}$$

$$a = \sqrt{\frac{\pi}{2}}$$

Aufgabe 5

Lineares Programm in normierter Form:

$$-2x_1 - 6x_2 \leq -12$$
$$x_1 \leq 6$$
$$x_2 \leq 6$$

$$x_1 \geq 0$$
$$x_2 \geq 0$$

$$\underbrace{-z}_{z^* \text{ max!}} \quad -x_1 + 3x_2 = -5$$

Simplex-Verfahren

x_1	x_2	x_3	x_4	x_5		
$\boxed{-2}$	-6	1	0	0	-12	$] :(-2)$
1	0	0	1	0	6	
0	1	0	0	1	6	
-1	3	0	0	0	-5	
$\boxed{1}$	3	$-1/2$	0	0	6	$\cdot(-1)$ +
1	0	0	1	0	6	+
0	1	0	0	1	6	
-1	3	0	0	0	-5	+

1	3	$-1/2$	0	0	6	
0	-3	$\boxed{1/2}$	1	0	0	$\frac{0}{1/2}=0$] $:1/2$
0	1	0	0	1	6	
0	6	$-1/2$	0	0	1	

1	3	$-1/2$	0	0	6	+
0	-6	$\boxed{1}$	2	0	0	$\cdot 1/2$
0	1	0	0	1	6	
0	6	$-1/2$	0	0	1	+

1	0	0	1	0	6
0	-6	1	2	0	0
0	1	0	0	1	6
0	3	0	1	0	1

Also

$x_1 = 6$

$x_2 = 0$

$z^* = 1$, d.h. $z = -1$