Aufgabe 1

Lagrange-Funktion:

$$Z(x, y, \lambda) = x^2 + y^2 - 4x - 6y + 14 + \lambda(3x + 2y - 6)$$

Damit

$$Z'_x = 2x - 4 + 3\lambda = 2x + 3\lambda - 4$$

$$Z'_y = 2y - 6 + 2\lambda = 2y + 2\lambda - 6$$

$$Z'_\lambda = 3x + 2y - 6 = 3x + 2y - 6$$

Nun $Z_x' = Z_y' = Z_\lambda' = 0$ (notwendige Bedingung)

Gaußsches Eliminationsverfahren

x	y	λ		
$\boxed{2}$	0	3	4	: 2
0	2	2	6	
3	2	0	6	
$\boxed{1}$	0	3/2	2	$\cdot(-3)$
0	2	2	6	
3	2	0	6	+

$$
\begin{array}{ccc|cll}
1 & 0 & 3/2 & 2 & & \\
0 & \boxed{2} & 2 & 6 & :2 & \\
0 & 2 & -9/2 & 0 & &
\end{array}
$$

$$
\begin{array}{ccc|cll}
1 & 0 & 3/2 & 2 & & \\
0 & \boxed{1} & 1 & 3 & \cdot(-2) & \\
0 & 2 & -9/2 & 0 & + &
\end{array}
$$

$$
\begin{array}{ccc|cll}
1 & 0 & 3/2 & 2 & & \\
0 & 1 & 1 & 3 & & \\
0 & 0 & \boxed{-13/2} & -6 & :\left(-\frac{13}{2}\right) &
\end{array}
$$

$$
\begin{array}{ccc|cll}
1 & 0 & 3/2 & 2 & + & \\
0 & 1 & 1 & 3 & & + \\
0 & 0 & \boxed{1} & 12/13 & \cdot\left(-3/2\right) & (-1)
\end{array}
$$

$$
\begin{array}{ccc|c}
1 & 0 & 0 & 8/13 \\
0 & 1 & 0 & 27/13 \\
0 & 0 & 1 & 12/13
\end{array}
$$

Also $x = \frac{8}{13}$ und $y = \frac{27}{13}$

Hinreichende Bedingung:

$$z''_{xx} = 2 \qquad z''_{xy} = 0 \qquad z''_{x\lambda} = 3$$

$$z''_{yy} = 2 \qquad z''_{y\lambda} = 2$$

$$z''_{\lambda\lambda} = 0$$

Also

$$D = \begin{vmatrix} 2_{+} & 0_{-} & 3_{+} \\ 0 & 2 & 2 \\ 3 & 2 & 0 \end{vmatrix}$$

$$= 2 \cdot \begin{vmatrix} 2 & 2 \\ 2 & 0 \end{vmatrix} + 3 \begin{vmatrix} 0 & 2 \\ 3 & 2 \end{vmatrix}$$

$$= 2(-4) + 3 \cdot (-6) = -26 < 0$$

Damit Minimum im Punkt

$$x = \frac{8}{13}$$

$$y = \frac{27}{13}$$

$$z = \left(\frac{8}{13}\right)^2 + \left(\frac{27}{13}\right)^2 - 4 \cdot \frac{8}{13} - 6 \cdot \frac{27}{13} + 14 = \frac{49}{13}$$

Aufgabe 2

x_1	x_2	x_3	x_4	x_5		
-1	2	1	0	0	10	
$\boxed{4}$	5	0	1	0	5	$\frac{5}{4}$] :4
1	0	0	0	1	5	$\frac{5}{1}$
-2	-1	0	0	0	0	
-1	2	1	0	0	10	+
$\boxed{1}$	$5/4$	0	$1/4$	0	$5/4$	+ (-1) 2
1	0	0	0	1	5	+
-2	-1	0	0	0	0	+
0	$13/4$	1	$1/4$	0	$45/4$	
1	$5/4$	0	$1/4$	0	$5/4$	
0	$-5/4$	0	$-1/4$	1	$15/4$	
0	$3/2$	0	$1/2$	0	$5/2$	

Also

$$x_1 = \frac{5}{4}$$

$$x_2 = 0$$

$$z = \frac{5}{2}$$

Aufgabe 3

Zunächst gilt

$$\frac{f(x+h)-f(x)}{h} = \frac{((x+h)^2+(x+h)+1)-(x^2+x+1)}{h}$$

$$= \frac{x^2+2xh+h^2+x+h+1-x^2-x-1}{h}$$

$$= \frac{2xh+h^2+h}{h} = 2x+h+1$$

$$= 2x+1+h$$

Also

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = 2x+1+\lim_{h\to 0} h$$

$$= 2x+1+0 = 2x+1$$

Aufgabe 4

$$\int 2\cos^2 x\,dx = 2\int \cos x \cdot \cos x\,dx$$

Partielle Integration:

$f'(x) = \cos x$, d.h. $f(x) = \sin x$

$g(x) = \cos x$, d.h.

Also

$$\ldots = 2\left(\sin x \cos x - \int \sin x(-\sin x)\,dx\right)$$

$$= 2 \sin x \cos x + 2\int \underbrace{\sin^2 x}_{1-\cos^2 x}\,dx$$

$$= 2 \sin x \cos x + 2\int (1-\cos^2 x)\,dx$$

$$= 2 \sin x \cos x + 2\int 1\,dx - 2\int \cos^2 x\,dx$$

$$= 2 \sin x \cos x + 2x - \int 2\cos^2 x\,dx$$

Also

$$2\int 2\cos^2 x\,dx = 2\sin x \cos x + 2x + C$$

womit

$$\int 2\cos^2 x\,dx = \sin x \cos x + x + C$$

Aufgabe 5

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Gaußsches Eliminationsverfahren

x_1	x_2	x_3		
0	3	1	2	
−1	−2	0	−5	+
$\boxed{1}$	5	1	7	+
0	3	$\boxed{1}$	2	(−1) (−1)
0	3	1	2	+
1	5	1	7	+
0	3	1	2	
0	0	0	0	XXX
1	2	0	5	
0	3	1	2	
1	2	0	5	

Also $x_1 = 5 - 2c$

$x_2 = c$ beliebig

$x_3 = 2 - 3c$

womit

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix} c$$