Aufgabe 1

Normierte Form:

$$-5x_1 + 2x_2 \leq 0$$
$$-x_1 + 2x_2 \leq 8$$
$$x_1 + x_2 \leq 10$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

2 $$-2x_2 = 0$$

Simplex-Verfahren:

x_1	x_2	x_3	x_4	x_5		
-5	$\boxed{2}$	1	0	0	0	$\frac{0}{2}=0$] : 2
-1	2	0	1	0	8	$\frac{8}{2}=4$
1	1	0	0	1	10	$\frac{10}{1}=10$
0	-2	0	0	0	0	
-5/2	$\boxed{1}$	1/2	0	0	0	$\cdot(-2)$ $\cdot(-1)$ $\cdot 2$
-1	2	0	1	0	8	+
1	1	0	0	1	10	+
0	-2	0	0	0	0	+

→

-5/2	1	1/2	0	0	0	
4	0	-1	1	0	8	$\frac{8}{4}=2$] :4
7/2	0	-1/2	0	1	10	$\frac{20}{7}$
-5	0	1	0	0	0	

-5/2	1	1/2	0	0	0	+
1	0	-1/4	1/4	0	2	·5/2 ·(-7/2) 5
7/2	0	-1/2	0	1	10	+
-5	0	1	0	0	0	+

0	1	-1/8	5/8	0	5	
1	0	-1/4	1/4	0	2	
0	0	**3/8**	-7/8	1	3	] :3/8
0	0	-1/4	5/4	0	10	

0	1	-1/8	5/8	0	5	+
1	0	-1/4	1/4	0	2	+
0	0	**1**	-7/3	8/3	8	·1/8 ·1/4 ·1/4
0	0	-1/4	5/4	0	10	+

0	1	0	1	1/3	6
1	0	0	-1/3	2/3	4
0	0	1	-7/3	8/3	8
0	0	0	2/3	2/3	12

Also

$x_1 = 4$

$x_2 = 6$

$z = 12$

Aufgabe 2

$$Z(x, y, \lambda) = 6x - 9 - x^2 - y^2 + \lambda(y - x - 1)$$

Notwendige Bedingung:

$$Z'_x = 6 - 2x - \lambda = 0$$

$$Z'_y = -2y + \lambda = 0$$

$$Z'_\lambda = y - x - 1 = 0$$

Aus $Z'_x + Z'_y = 0$ folgt

$$6 - 2x - 2y = 0$$

Und wegen $2 Z'_\lambda = 0$

$$2y - 2x - 2 = 0$$

Also

$$(6 - 2x - 2y) + (2y - 2x - 2) = 0$$

$$-4x + 4 = 0$$

$$-4x = -4$$

$$x = 1$$

und

$$y = x + 1 = -1 + 1 = 2$$

Hinreichende Bedingung:

$$z''_{xx} = -2$$

$$z''_{xy} = 0$$

$$z''_{x\lambda} = -1$$

$$z''_{yy} = -2$$

$$z''_{y\lambda} = 1$$

$$z''_{\lambda\lambda} = 0$$

Also

$$D = \begin{vmatrix} -2 & 0 & -1 \\ 0 & -2 & \boxed{1} \\ -1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} -2 & -2 & 0 \\ 0 & -2 & \boxed{1} \\ -1 & 1 & 0 \end{vmatrix}$$

$$= (-1) \begin{vmatrix} -2 & -2 \\ -1 & 1 \end{vmatrix} = (-1)(-2-2) = 4 > 0$$

→

$$z = 6 \cdot 1 - 9 - 1^2 - 2^2$$

$$= 6 - 9 - 1 - 4 = -8$$

Damit Maximum im Punkt

$$(1 \mid 2 \mid -8)$$

Aufgabe 3

Es ist ein lineares System mit 4 Gleichungen und 3 Variablen zu lösen.

Normierte Form:

$$\begin{array}{rrrcr} x_1 & & -x_3 & = & -7 \\ & x_2 & +x_3 & = & 4 \\ -x_1 & -x_2 & & = & 3 \\ x_1 & & +x_3 & = & 3 \end{array}$$

Gaußsches Eliminationsverfahren:

x_1	x_2	x_3			
$\boxed{1}$	0	-1	-7	+	$\cdot(-1)$
0	1	1	4		
-1	-1	0	3	+	
1	0	1	3		+

x_1	x_2	x_3		
1	0	-1	-7	
0	$\boxed{1}$	1	4	+
0	-1	-1	-4	+
0	0	2	10	

→

$$
\begin{array}{ccc|cll}
\hline
1 & 0 & -1 & -7 & & \\
0 & 1 & 1 & 4 & & \\
\cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & & \\
0 & 0 & \boxed{2} & 10 & :2 & \\
\hline
1 & 0 & -1 & -7 & + & \\
0 & 1 & 1 & 4 & & + \\
0 & 0 & \boxed{1} & 5 & + & \cdot(-1) \\
\hline
1 & 0 & 0 & -2 & & \\
0 & 1 & 0 & -1 & & \\
0 & 0 & 1 & 5 & & \\
\hline
\end{array}
$$

Also

$x_1 = -2$

$x_2 = -1$

$x_3 = 5$

Aufgabe 4 Partielle Integration

$$\int \underbrace{x^2}_{g} \underbrace{\cos x}_{f'} \, dx$$

$$= \underbrace{\sin x}_{f} \; \underbrace{x^2}_{g} - \int \underbrace{\sin x}_{f} \; \underbrace{2x}_{g'} \, dx$$

$$= x^2 \sin x - 2 \int \underbrace{x}_{g} \; \underbrace{\sin x}_{f'} \, dx$$

$$= x^2 \sin x - 2 \left(\underbrace{(-\cos x)}_{f} \; \underbrace{x}_{g} - \int \underbrace{(-\cos x)}_{f} \cdot \underbrace{1}_{g'} \, dx \right)$$

$$= x^2 \sin x + 2x \cos x - 2 \sin x + C$$

Aufgabe 5

$$f(x) = e^{2x}$$

$$f'(x) = 2e^{2x}$$

$$f''(x) = 2 \cdot 2 e^{2x}$$

$$f'''(x) = \underbrace{2 \cdot 2 \cdot 2}_{3 \text{ Mal}} e^{2x}$$

usw.

$$f^{(k)}(x) = \underbrace{2 \cdot \ldots \cdot 2}_{k \text{ Mal}} e^{2x} = 2^k e^{2x}$$

Also

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k$$

$$= \sum_{k=0}^{\infty} \frac{2^k e^{2 \cdot 0}}{k!}(x-0)^k$$

$$= \sum_{k=0}^{\infty} \frac{2^k}{k!} x^k$$