Aufgabe 1

$$z(x, y, \lambda) = 4xy^2 - x^2y - 4x + \lambda(x-y)$$

Notwendige Bedingung:

$$z_x' = 4y^2 - 2xy - 4 + \lambda = 0$$

$$z_y' = 8xy - x^2 - \lambda = 0$$

$$z_\lambda' = x - y = 0$$

Also $x = y$ und so

$$z_x' + z_y' = 4x^2 - 2x^2 - 4 + 8x^2 - x^2 \underbrace{+\lambda - \lambda}_{=0} = 0$$

$$9x^2 - 4 = 0$$

$$9 \cdot x^2 = 4$$

$$x^2 = \frac{4}{9} = \left(\frac{2}{3}\right)^2$$

womit

$$x = \pm \frac{2}{3} \quad \text{und} \quad y = \pm \frac{2}{3}$$

$\rightarrow$

Hinreichende Bedingung:

$$z''_{xx} = -2y$$

$$z''_{xy} = 8y - 2x$$

$$z''_{x\lambda} = 1$$

$$z''_{yy} = 8x$$

$$z''_{y\lambda} = -1$$

$$z''_{\lambda\lambda} = 0$$

Für $x = y = \frac{2}{3}$

$$D = \begin{vmatrix} -\frac{4}{3} & 4 & \boxed{1} \\ 4 & \frac{16}{3} & -1 \\ 1 & -1 & 0 \end{vmatrix} \begin{matrix} + \\ + \\ \\ \end{matrix}$$

$$= \begin{vmatrix} -\frac{4}{3} & 4 & 1 \\ \frac{8}{3} & \frac{28}{3} & 0 \\ 1 & -1 & 0 \end{vmatrix}$$

→

$$= 1 \cdot \begin{vmatrix} \frac{8}{3} & \frac{28}{3} \\ 1 & -1 \end{vmatrix}$$

$$= -\frac{8}{3} - \frac{28}{3} = -12 < 0$$

$$z = 4\left(\frac{2}{3}\right)^3 - \left(\frac{2}{3}\right)^3 - 4\,\frac{2}{3}$$

$$= 3\left(\frac{2}{3}\right)^3 - \frac{8}{3}$$

$$= \frac{8}{9} - \frac{8}{3} = -\frac{16}{9}$$

Also Minimum im Punkt

$$\left(\frac{2}{3} \;\middle|\; \frac{2}{3} \;\middle|\; -\frac{16}{9}\right)$$

Aufgabe 2

Es handelt sich um ein Gleichungssystem mit 3 Variablen und 4 Gleichungen.

Normierte Form:

$$
\begin{aligned}
x_1 \phantom{{}+x_2} &+ x_3 = 4 \\
-x_1 + x_2 \phantom{{}+x_3} &= -5 \\
x_2 &- x_3 = -3 \\
x_1 \phantom{{}+x_2} &+ x_3 = 4
\end{aligned}
$$

Gaußsches Eliminationsverfahren:

x_1	x_2	x_3		
$\boxed{1}$	0	1	4	+ ·(−1)
−1	1	0	−5	+
0	1	−1	−3	
1	0	1	4	+
1	0	1	4	
0	$\boxed{1}$	1	−1	·(−1)
0	1	−1	−3	+
~~0~~	~~0~~	~~0~~	~~0~~	

→

$$\begin{array}{ccc|c|l}
1 & 0 & 1 & 4 & \\
0 & 1 & 1 & -1 & \\
0 & 0 & \boxed{-2} & -2 & :(-2) \\
\hline
1 & 0 & 1 & 4 & + \\
0 & 1 & 1 & -1 & + \\
0 & 0 & \boxed{1} & 1 & \cdot(-1) \\
\hline
1 & 0 & 0 & 3 & \\
0 & 1 & 0 & -2 & \\
0 & 0 & 1 & 1 &
\end{array}$$

Also

$x_1 = 3$

$x_2 = -2$

$x_3 = 1$

<u>Aufgabe 3</u>

Normierte Form:

$$-5x_1 + 2x_2 \leq 0$$
$$-x_1 + 2x_2 \leq 8$$
$$x_1 + x_2 \leq 10$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

$$\underbrace{-z}_{z^*} + 6x_1 - 4x_2 = 0$$

Simplex-Verfahren:

x_1	x_2	x_3	x_4	x_5			
-5	$\boxed{2}$	1	0	0	0	$\frac{0}{2}=0$	: 2
-1	2	0	1	0	8	$\frac{8}{2}=4$	
1	1	0	0	1	10	$\frac{10}{1}=10$	
6	-4	0	0	0	0		
-5/2	$\boxed{1}$	1/2	0	0	0	·(-2) ·(-1) ·4	
-1	2	0	1	0	8	+	
1	1	0	0	1	10	+	
6	-4	0	0	0	0	+	

→

-5/2	1	1/2	0	0	0	
$\boxed{4}$	0	-1	1	0	8	$\frac{8}{4}=2$] : 4
7/2	0	-1/2	0	1	10	$\frac{20}{7}$
$\boxed{-4}$	0	2	0	0	0	

-5/2	1	1/2	0	0	0	+
$\boxed{1}$	0	-1/4	1/4	0	2	$\cdot 5/2$ $(-7/2)$ $\cdot 4$
7/2	0	-1/2	0	1	10	+
-4	0	2	0	0	0	+

0	1	-1/8	5/8	0	5
1	0	-1/4	1/4	0	2
0	0	3/8	-7/8	1	3
0	0	1	1	0	8

Also

$x_1 = 2$

$x_2 = 5$

$-z = 8$, d.h. $z = -8$

Aufgabe 4

$$f(x) = e^x$$

$$f'(x) = e^x$$

$$f''(x) = e^x$$

usw.

$$f^{(k)}(x) = e^x$$

Also

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!} (x - x_0)^k$$

$$= \sum_{k=0}^{\infty} \frac{e^3}{k!} (x-3)^k$$

Aufgabe 5 Partielle Integration

$$\int \underset{g}{x^2}\, \underset{f'}{e^{2x}}\, dx$$

$$= \underbrace{\tfrac{1}{2}e^{2x}}_{f}\, \underbrace{x^2}_{g} - \int \underbrace{\tfrac{1}{2}e^{2x}}_{f}\, \underbrace{2x}_{g'}\, dx$$

$$= \frac{1}{2}x^2 e^{2x} - \int \underset{g}{x}\, \underset{f'}{e^{2x}}\, dx$$

$$= \frac{1}{2}x^2 e^{2x} - \left(\underbrace{\tfrac{1}{2}e^{2x}}_{f}\, \underbrace{x}_{g} - \int \underbrace{\tfrac{1}{2}e^{2x}}_{f} \cdot \underset{g'}{1}\, dx \right)$$

$$= \frac{1}{2}x^2 e^{2x} - x\frac{1}{2}e^{2x} + \frac{1}{4}e^{2x} + C$$

$\rightarrow$

$$= \frac{1}{2} e^{2x} \left(x^2 - x + \frac{1}{2} \right) + C$$